*Real Competition within an Industry under Technical Change*

# Classical-Evolutionary Dynamics of Price Formation

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**Abstract**

This paper presents a computational model of probabilistic price competition among many firms within a single industry with demand slowly flowing from firms with high to low prices and constant production costs. We show that price gravitation around a well-defined industry competitive price can be easily reproduced following the classical framework of real competition assuming minimal rationality on the economic agents, maximum parsimony in the model and no recourse to optimization whatsoever. Decentralized, random price-setting by firms under demand pressures induces interdependent stochastic growth in their capital shares regulated by their profit rates and ultimately by their cost structure, while yielding an emergent classical price of production or neoclassical competitive price as an attractor of stable oscillations. The computational model also provides a classical mechanism to reproduce a spectrum of profit rate differentials based on evolving, heterogeneous costs among firms. Fundamental assumptions in economic theory such as profit maximization and equilibrium are addressed.

**JEL Codes** B51, B52, C63, D20, D43, E11, E14

## 1 Introduction

The assumption of profit maximization as guiding principle in firm behavior, by which firms generally set prices where their marginal revenue equals their marginal cost, is standard in economic theory, in the perfectly competitive framework of general equilibrium and imperfect oligopolistic competition alike. The assumption is very useful to the extent that it allows to employ the conventional marginalist techniques of mathematical optimization in the analysis of economic behavior. However, the justification of such a widely accepted assumption has stood on flimsy grounds in the light of the actual behavior of firms as related by the empirical evidence in the business literature, a criticism that started as soon as the 1930s with the study of Gardiner Means on administered prices and the findings by Hall and Hitch of the Oxford Economists Research

Group by which firms used full-cost pricing. For Herbert Simon, firms 'satisfice' by using simple rules of thumb to search for available alternatives until a threshold is met because their rationality is in fact bounded by information-gathering costs [Simon, 1956]. For Alchian, 'profit maximization' in the context of fundamental uncertainty is in fact "meaningless as a guide to specifiable action" [Alchian, 1950]. In an early paper, Gary Becker argues that it is the market that behaves rationally, while individual behavior only needs to be subject to budgetary constraints even if it is irrational [Becker, 1962].

Following the famous methodological argument by Milton Friedman, neoclassical theorists usually respond to this contradictory evidence arguing that although the assumption is clearly unrealistic in terms of actual behavior, it does not actually require firms to perform optimizing calculations in reality, but just that they behave 'as if' they did [Friedman, 1953]. In his 'selectionist' argument that is typical of the Chicago school, Friedman uses an analogy with a snooker player that does not need to calculate the trajectories of the balls following the physical laws of mechanics in order to win a competition, but only needs to behave "as if he did". Yet "as-if" claims are methodologically problematic since they are very difficult to demonstrate empirically. In his *Theory of Industrial Organization*, Jean Tirole lays out a similar selectionist argument in the words of Scherer [Tirole, 1988]:

> *When forced into the trenches on the question of whether firms maximize profits, economists resort to the ultimate weapon in their arsenal: a variant of Darwin's natural selection theory. Over the long pull, there is one simple criterion for the survival of a business enterprise: Profits must be nonnegative. No matter how strongly managers prefer to pursue other objectives, and no matter how difficult it is to find profit-maximizing strategies in a world of uncertainty and high information costs, failure to satisfy this criterion means ultimately that a firm will disappear from the economic scene.* [Scherer, 1980, p.38]

This is faulty logic. Profits must be nonnegative indeed, but it simply does not follow that firms are profit-maximizing as neoclassical theory states, but only profit-seeking – in more classical and post-Keynesian lines. Alchian already emphasized that it is the requirement of realized positive profit, not maximum profit, that acts as the survival filter for firms and its mark of success and viability, regardless of their motivation or process of reasoning to reach such result. For P.W.S. Andrews and authors in the post-Keynesian tradition such as Lee, the firm is an organic entity, "living" in historical time, whose ultimate objectives are survival and, if possible, growth [Andrews, 1964; Lee, 1999]. Selectionist arguments in favor of profit maximization may be flawed since they impute to reality an optimality that may not be warranted, as cautioned against by evolutionary theorists Jay Gould and Lewontin in their seminal paper *The Spandrels of San Marco.* It thus seems that the mere purpose of retaining profit maximization –which underlies the assumption of economic equilibrium–

as main causal mechanism in economic theory is for the convenience of formal analytical analysis.

The purpose of this paper is to target and clarify the selectionist argument by highlighting the crucial difference between profit-maximizing and profit-seeking behavior in firms. For that matter, we develop a simple computational model of decentralized price competition within a single industry based on the classical theory of real competition of Anwar Shaikh [2016] that can be understood as a mechanism of evolutionary selection of techniques of production, where the dimension of time and a classical conception of the profit rate are essential in the analysis. We show that mathematical analysis of firm behavior is possible without any recourse to optimization and still generate an emergent competitive price on the basis of the classical causal mechanism of reproduction, yielding results that can be empirically tested. Instead of imposing *ad hoc* the conventional assumption of equilibrium as a single-point, static rest state as the logical outcome of optimizing agents' decisions, we obtain a statistical equilibrium as the robust, emergent outcome of firms' probabilistic decisions [Simon and Bonini, 1958; Mirowski, 1989; Drăgulescu and Yakovenko, 2001; Scharfenaker and Foley, 2017].

## 2 On Theories of Competition

In the neoclassical literature, Walrasian general equilibrium offers a peculiar story of price formation by resorting to a fictitious Walrasian auctioneer, passive price-taking, profit-maximizing firms, while banning trade at nonequilibrium prices [Fisher, 1989; Roberts, 1991]. The neoclassical theory of perfect competition portrays competition within an industry in terms of a very large number of very small firms that are identical in scale and cost structure, with the same vanishingly small market share, all facing the same horizontal demand curve (so they can sell all that they choose to produce), and passive with respect to price and technology. In the context of game theory, a wide catalogue of models focusing on price-setting oligopolies exist on different institutional arrangements, while preserving some degree of profit-maximizing rationality, and examine how perfectly competitive outcomes can be retrieved [Levitan and Shubik, 1972; Benassy, 1976; Hart, 1982; Allen and Hellwig, 1986; Dixon, 1987; Maskin and Tirole, 1988]. Output-setting Cournot models of oligopoly yielding a Nash equilibrium have also been extensively studied, while preserving the existence of a Walrasian auctioneer [Allen and Hellwig, 1986]. An early pre-Walrasian model of price-setting competition is the duopoly discussed by Bertrand where a static weak Nash equilibrium is reached and the competitive price is charged. In Bertrand's model, the firm charging the higher price loses all demand to the firm with the lower price that takes the whole market, thus inducing them to undercut prices to the marginal cost, below which they would have negative profit. In his review of Bertrand, Edgeworth remarked that in presence of capacity constraints no equilibrium point could be reached, inducing price oscillations [Maskin and Tirole, 1988; Noel, 2011].

However, as early as in the 1930s the Hall and Hitch report of the Oxford Economists Research Group crucially showed empirical evidence of actual business practices that firms focused more on price-setting rather than output-setting (in contrast to Walrasian general equilibrium) and their price-setting mechanism had more characteristics of full-cost pricing than the conventional marginalist rule [**?**]. Since in perfect competition firms are passive price-takers, this came to be regarded as an outcome of oligopolistic, "imperfect" behavior, where markups over prices had to be seen as the result of price-setting market power [Chamberlin, 1949; Robinson, 1969]. Cost-plus pricing was also empirically supported by Gardiner Means' early research on administered prices and became the basis for post-Keynesian markup-pricing theory on Kaleckian lines [Lee, 1999]. Unfortunately, post-Keynesian theory does not offer a robust explanation of what actually defines the markup, but only tentatively suggests a wide range of possible sources (monopoly power, degree of concentration, risk of new entry, class struggle, target rates of return) rather than adopting the cost-based framework of the classical structural theories of relative prices [Shaikh, 2016, pp.361-362]. In 1984, Semmler examined the empirical predictions of different theoretical models of competition –perfect/neoclassical, imperfect/oligopolist, and classical/Marxian– and concluded only the framework provided by the latter was empirically consistent with the actual data – the markup being related to profitability [Semmler, 1984].

In the classical view, competition is the direct result of the self-expansion of capital [Semmler, 1981, p.41]. Competition between capitals does not bring a smooth process of adjustment and convergence towards equilibrium prices and quantities, but disequilibria and long-lasting deviations from the center of gravity [Semmler, 1981, p.41]. In the classical framework of Shaikh's real competition, firms are viewed as active, competitive, profit-driven, ever-expanding, price-setting, and cost-cutting. Unlike the theories of perfect and oligopolistic competition, Shaikh's framework presents a well-defined structure of profit rates within and between industries that is consistent with empirical observation [Shaikh, 2016]. The neoclassical concept of equilibrium as a Marshallian achieved rest state is replaced by the more realistic classical notion of competition as a process of dynamical turbulent equilibration, defining oscillatory trajectories of market prices around values as statistical centers of gravity [Harris, 1988]. Under real competition, turbulent price equalization within an industry and profit rate equalization between industries are quintessential emergent properties [Shaikh, 2016]. Free capital mobility from low- to high-profitability lines of production turbulently equalizes the profit rates of price-leaders over industry-specific cycles. Within an industry, prices set by competing firms are roughly equalized over demand flowing from higher to lower-price firms. This induces the "law of correlated prices" as opposed to the neoclassical "law of one price", which disequalizes profit rates and margins depending on the cost spectrum within an industry [Shaikh, 2016, p.262]. The price leader embodies the best generally reproducible condition of production, with the lowest reproducible (quality-adjusted) costs in the industry, that is, the regulating capital [Shaikh, 2016, p.265].

Most classical models study the evolution of a multi-sectorial economy of the Leontief input-output form on the basis of cross-dual adjustments between prices and quantities yielding convergence to long-run equilibrium prices, for instance the seminal work by Flaschel and Semmler [1987; 1990] or Duménil and Lévy [1993], as well as later agent-based models [Wright, 2008, 2011; Cogliano, 2012]. Cross-dual adjustments tend to be based on reaction functions between prices and quantities regulated by a reaction parameter, which is very difficult to estimate empirically. Jiang's study found endogenous cycles and chaos on a disaggregated version of Foley's circuit-of-capital model that critically depended on a homogeneous structure of the capital outlays-sales matrix between the firms, which did not look at the dynamics of price competition itself [Foley, 1982; Jiang, 2015].

# 3 A classical-evolutionary model of price competition

In this paper, we study computer simulations of decentralized price competition within a single industry (i.e. selling an undifferentiated homogeneous product) among price-setting firms with minimal rationality and no recourse to any optimization, but constrained by reproduction. The classical view of a microeconomics of the firm uses Marx's general formula of capital as the starting point of the analysis with capitals as basic self-expanding evolutionary agents, as autonomous individual parts of the total social capital. We use a pure-circulating capital model, i.e. capital that is fully employed in one production period under constant average costs. We strive for maximum parsimony in the model specification. In the classical view, profit drives investment [Ruggles, 1992] and thus regulates microeconomic growth – hence profitability can be understood as a measure of evolutionary fitness. Under constant production costs, the dynamics will lie in price-setting competition, where self-expanding firms strive to balance the impact on profits (and thus growth) of the trade-off between interdependent changes in price and sales volume under the aggregate constraint of constant demand.

Following Alchian's evolutionary approach rejecting rational optimization and in contrast to the cross-dual literature, firms set prices entirely by trial and error in blind attempts to undercut its competitors while preserving its reproduction in the market – that is, actual *tâtonnement*: "to search or attempt to find something in the dark, or, as a blind person, by feeling; to move about hesitatingly, as in darkness or obscurity; to feel one's way, as with the hands, when one can not see"[1]. Empirical research in the business literature lists up to 19 different pricing strategies, with most firms employing 3 or 4 of them [Rao, 2009, pp.18-19]. It is rather more cavalier to assume minimal rationality on firm behavior by making price-setting probabilistic under simple rules of thumb, instead of imposing very specific behavior in their pricing strategies. The research

[1] Wiktionary definition.

question thus becomes what minimal assumptions are required in the model in addition to simple accounting identities so that stable price equalization can emerge at the industry level around a competitive price of production. The trick just consists of setting the individual profit rates as dependent on the deviation of the firm's price from the average (hence heterogeneous prices are essential in the model): if a firm underprices its competitors, it will gain market share and thus obtain a higher profit rate. However, we do not impose any requirement on the price average to obtain the competitive price and let firms find it by themselves – we abstain from imposing the conventional assumption of economic equilibrium.

Building on the classical framework of real competition of Anwar Shaikh [2016], we find that a classical price of production as a center of gravity (only defined by the lowest cost in the industry and the regulating profit rate) still emerges when imposing two particular assumptions on demand: demand is fixed in monetary terms (i.e. a constant total mass of revenue, since we only focus on price competition) but it also flows from high to low prices (in a dynamical version of the downward-sloping demand curve). The exogenous parameters in the model are the number of firms $N$ in the industry, the total capital of the industry $M$, the normal profit rate $r$, and the cost structure of firms (a vector $\vec{ac}$ of size $N$). Firms are assumed to operate uniquely on circulating capital financing its investment out of past sales in a scenario of simple reproduction. We abstract from the credit market.

## 3.1 Accounting identities

Following Marx's general formula [Marx, 1976, ch.4], the starting point in our model is the circuit of capital as a micro-economic model of the firm as self-expanding value:

$$M - C - M' \tag{1}$$

At the single industry level, the total capital of the industry is $M$, which produces output $C'$ from input $C$ to sell in the marketplace with profit $\Pi = rM$. In the scenario of simple reproduction, the same capital $M$ will be re-committed to production in the next time period, while profit $\Pi$ goes to consumption:

$$M - \underbrace{C}_{\text{input}} \ldots P \ldots \underbrace{C'}_{\text{output}} - M' = (1+r)M \begin{cases} M & \text{for production} \\ \Pi = rM & \text{for consumption} \end{cases} \tag{2}$$

For the sake of convenience, all capital is circulating, i.e. it is fully employed after only one production period. The total capital of the industry is $M$. In the scenario of simple reproduction, the profit rate of the industry is equal to the normal profit rate of the economy, so the total mass of capital is always constant at $M$ over time. A constant mass of profit is proportional to the total capital of the industry,

$$\Pi = M' - M = rM \tag{3}$$

where $r$ is the normal profit rate, an exogenous parameter altogether with the size of industry $M$ and the cost structure. This fixes industry demand in monetary terms, which is the total mass of revenue, $M' = (1 + r)M$.

The total mass of capital can now be divided into particular firms as individual fractions of the total social capital, $i = 1, \ldots, N$, producing a homogeneous commodity $C'$ with production average costs $ac_i$, with corresponding individual capitals $M_i$, such that total capital remains $M = \sum_{i=1}^{N} M_i$. We define the capital share or frequency of the $i$th firm as

$$x_i = \frac{M_i}{M} \tag{4}$$

so that $\sum_{i=1}^{N} x_i = 1$. Each individual capital has revenue $M_i' = M_i + \pi_i = M_i + r_i M_i = (1 + r_i)M_i$. Fixing the total mass of revenue is equivalent to the weighted average of individual profit rates $r_i$ yielding the aggregate profit rate $r$,[2]

$$r = \sum_{i=1}^{N} x_i r_i \tag{5}$$

This equation expresses that what one firm loses in total profit the other gains, with constant total revenue $M' = (1 + r)M$ and total supply $X = \sum_{i}^{N} \frac{x_i M}{ac_i}$, where $x_i$ and $ac_i$ correspond to the capital share and constant average costs, respectively.

## 3.2 Time evolution

In our scenario of simple reproduction, no surplus is recapitalized in the aggregate (i.e. the mass of capital $M$ is constant as in Wright [2008]), but only re-distributed between the firms within an industry so that we may have expanded and contracted reproduction at the micro-level. In a classical context, the profit rate regulates the time evolution of the capital share of the firm and thus operates as a well-defined evolutionary fitness function. In the classical view, profits drive investment: investment is mostly financed out of corporate retained earnings, so that the business savings rate tracks very closely the investment rate [Ruggles, 1992]. If a firm happens to have a higher profit rate than the normal profit rate, it will expand in the next timestep. If it is lower, it will contract.

We formalize this by stating that the capital share at the next timestep $x_i(t + 1)$ must be equal to the $i$th share of the total revenue after sales have been realized, $M_i'/M'$:

$$x_i(t+1) = \frac{M_i'(t)}{M'} = \frac{(1 + r_i)M_i(t)}{(1 + r)M} = \frac{1 + r_i}{1 + r} x_i(t) \tag{6}$$

[2] *Proof* $(1 + r)M = \sum_{i=1}^{N}(1 + r_i)M_i = M + \sum_{i=1}^{N} r_i M_i$

$$r = \sum_{i=1}^{N} r_i \frac{M_i}{M} = \sum_{i=1}^{N} r_i x_i$$

yielding the time evolution rule over the capital share $x_i$, which has only a fixed point ($x_i(t+1) = x_i(t)$) when $r_i = r$. If a dynamical-systems formulation is adopted, we have $\vec{x}(t+1) = A\vec{x}(t)$ where $A$ is a diagonal matrix such that $a_{ij} = \frac{1+r_i}{1+r}$ for $i = j$ and 0 if otherwise.

Equation (6) implies that if $r_i > r$ the firm adds a share $\sigma_i$ of the surplus to its capital stock of the next timestep, while the remainder goes to consumption. In order to see this, we can write the identity

$$M_i(t+1) = M_i(t) + \sigma_i \Delta M_i(t) = (1 + \sigma_i r_i) M_i(t) \tag{7}$$

where $\Delta M_i(t) = r_i M_i(t)$ are the profits made selling the output produced and $\sigma_i$ is the individual rate of its recapitalization back into the circuit of capital of the next iteration $t+1$. We can compute $\sigma_i$ from (6) to be:[3]

$$\sigma_i = \frac{1 - r/r_i}{1 + r} = \frac{r_i - r}{r_i(1 + r)} \tag{8}$$

Using the Cambridge savings equation, we can also write the individual growth rate as:

$$g_i(t) = \sigma_i r_i = \frac{r_i - r}{1 + r} \tag{9}$$

If the profit rate is equal to the normal profit rate, i.e. $r_i = r$, then the firm gains the normal share of total profits and initiates the next step with same capital share and amount of capital, so that $\sigma_i = 0$. If the profit rate is higher than normal, the firm appropriates a higher-than-normal share of total profits $rM$ so it increases its capital share $x_i$ at the expense of their competitors so that $\sigma_i$ is positive. $\sigma_i$ is negative if $r_i < r$ and the firm earns a lower-than-normal share of total profits, contracting in the next time period. This is the fundamental evolutionary mechanism at play.

The individual profit rate is what regulates the growth of the capital share in (6). By accounting definition, the profit rate of an individual capital $M_i$, producing output $X_i$ at average costs $ac_i$ and selling an amount $S_i$ at price $p_i$ is

$$r_i = \frac{M_i' - M_i}{M_i} = \frac{p_i S_i}{ac_i X_i} - 1 \tag{10}$$

Revenues made on the sale, $M_i' = p_i S_i$, depend on the decentralized, but interdependent process of price competition between firms under the downward pressures of demand, while output produced is determined by the capital stock $M_i$ and costs $ac_i$. The stochastic process operates subject to the constraint of invariance of the total mass of revenue with respect to total capital $M$ and normal profit rate $r$, $(1 + r)M$, which are exogenous parameters. This ensures the gravitation of individual profit rates over the normal profit rate over time and thus the stability of the growth paths of the capital shares.

[3] *Proof* Since $M$ is fixed, $M_i(t) = x_i(t)M$, so that $1 + \sigma_i r_i = \frac{1+r_i}{1+r}$. One obtains (8) after re-arranging to isolate $\sigma_i$.

## 3.3 Demand

At each timestep, firms first produce output $X_i$, as determined by $X_i = M_i/ac_i$, i.e. by constant average costs $ac_i$ and capital stock available $M_i$. The particular character of the model lies in its specification of demand in order to compute output sold $S_i$ in terms of the output produced $X_i$ and the own price deviation from the average,

$$\delta_i \equiv \langle p \rangle - p_i \tag{11}$$

where $\langle p \rangle$ is the average of firm prices (weighted by the capital share). In our dynamic characterization of the downward-sloping demand curve, if firm prices are heterogeneous, demand flows from the firms with higher prices to the firms with lower prices so that output sold $S_i$ will re-distribute output produced $X_i$ among firms depending on their trial-and-error prices with respect to the average. This is the crucial assumption in the model, which is a realistic economic fact: customers always opt for lower prices of the same product. If we consider the average price to be weighted, we can also view the price deviation of the $i$th firm as the weighted sum of all the price deviations of competing firms within the industry with respect to the $i$th firm's price:

$$\delta_i = \sum_{j=1}^{N} x_j (p_j - p_i) \tag{12}$$

Probabilistic price-setting makes the feedback effect of prices on profits ambiguous, since a price cut may not attract enough volume in sales to offset the decrease in the profit, while keeping total revenue constant. In order to formalize this, we write the ratio of output sold to output produced to be of the following functional form:

$$\frac{S_i}{X_i} = [1 + f(\delta_i)]\mu \tag{13}$$

The gap between output sold and produced will be then added to the firm's inventory,

$$inv_i(t+1) = inv_i(t) + X_i - S_i \tag{14}$$

$f(\delta_i)$ is conceived as a drift term that re-distributes output produced and sold in terms of price deviations from the average, while $\mu$ is a scaling factor on the $N$-vector $S$ in order to maintain the constancy of the total mass of revenue. Hence we impose the following requirements on $f(\delta_i)$ and $\mu$:

- If there is no price deviation, there is no drift: $f(0) = 0$
- The drift term must be increasing with respect to price deviation $\delta_i$: $f'(\delta_i) > 0$.
- Scaling factor $\mu$ must be so to fix demand in monetary terms, i.e. $\sum_i^N p_i S_i = (1 + r)M$

We can thus obtain the scaling factor $\mu$ by replacing $S_i$ into the above accounting identity for a constant demand:

$$\sum_{i=1}^{N} p_i S_i = \sum_{i=1}^{N} p_i X_i[1 + f(\delta_i)]\mu = \mu \sum_{i=1}^{N} p_i X_i[1 + f(\delta_i)] = (1 + r)M$$

$$\mu = \frac{(1 + r)M}{\sum_{i=1}^{N} p_i X_i[1 + f(\delta_i)]} = \frac{1 + r}{\sum_{i=1}^{N} p_i \frac{x_i}{ac_i}[1 + f(\delta_i)]} \tag{15}$$

where we used $X_i = x_i M/ac_i$. The scaling factor $\mu$ is related to the elasticity of demand to the average industry price and is necessary so that the weighted average of the individual profit rates is $r$.

The average capital share over capitals $i$ is $\langle x_i(t)\rangle = \frac{\sum_{i=1}^{N} x_i(t)}{N} = \frac{1}{N}$. With respect to the drift term $f(\delta_i)$, the most straightforward candidate function for $f(\delta_i)$ is the identity function, which characterizes the drift term as linear:

$$f(\delta_i) = \delta_i = \langle p\rangle - p_i \tag{16}$$

The drift term can also take a cubic form, $f(\delta_i) = \alpha\delta_i^3$, where $\alpha$ is a calibration parameter. This allows to explore many functional forms of demand. An alternative formulation that may be more elegant just defines the individual profit rate as $r_i = r_e q(1+f(\delta))$ and output sold $S_i = \frac{(1+r_i)M_i}{p_i}$, yielding similar results.

## 3.4 Pricing

In the classical theory of real competition, firms are crucially not price-takers as in perfect competition, but price-setters that engage in actual *tâtonnement*: they just test prices by trial and error in a decentralized fashion with minimal rationality and no recourse to optimization, in a way much closer to Herbert Simon's 'satisficing' style of decision-making or Alchian's observations of firm behavior under uncertainty. Our model of minimal rationality of firm behavior consists of defining two simple rules of thumb that trigger either a stochastic decrease or increase of the price in reaction to the firm's own state vector. The probabilistic nature of the price-setting process makes the individual profit rates regulating the growth of capital shares fundamentally stochastic as well as interdependent. Price is the action variable of firms, operating over a single dimension, over which firms make steps of size 0.05 units with a Gaussian error of 0.01. The rationale to model pricing decisions by firms in this way is to capture the over- and undershooting that occurs in the form of turbulent regulation within the classical theory of real competition.

Only two decision-making rules are specified, one for decreasing the price and another for increasing the price:

- **price-cut rule** firms lower their price enacting a price cut with uniform probability of 15% in order to attract demand or in the case they are losing capital share to other price-cutters (i.e. $x_i(t) < x_i(t-1)$). This rule reflects that pricing objectives of maintaining or increasing market share and sales volume are the most frequent among business practice [Rao, 2009, p.20]

- **price-increase rule** firms raise the price in case their average output sold has been greater than 1.005 times their average output produced. This rule reflects the next most frequent pricing objectives among business practice: increase or maintain sales revenue and money gross profit [Rao, 2009, p.20].

Since they are probabilistic, other behavioral rules can be specified with no significant change on the simulations (apart from the particular trajectory that the capital shares follow), for instance increasing the price if the inventory/sales ratio is above a threshold and decreasing it if it is below another. Other rules can be specified in order to explore to what extent the emergent macroeconomic patterns that are observed are 'robustly insensitive' to the particular specifications of micro-founded behavior. The rationale of the price-increase rule is for the firm not to be incentivized to decrease the price so much (to zero) that inventory is eventually depleted before new production can be added, induced by the capacity constraints imposed by aggregate simple reproduction.

Firms are killed if capital share is below $0.01/N$. In order to preserve the assumption of simple reproduction, we can assume new firms enter the market by taking over bankrupt ones, while keeping the capital share but copying the cost structure of the regulating capital, reflecting new investment in the industry. Another modeling option is to assume other existing firms take over the bankrupt ones, reflecting the centralization of capital within an industry [Marx, 1976, ch.25].

# 4 Simulation Results

At any instant of time $t$, the system is represented by a $N \times 7$ state matrix containing microeconomic variables capital $M$, unit average costs $ac$, inventory $inv$, price $p$, profit rate $r$, capital share $x$, output produced $X$ and output sold $S$ for each individual firm $i = 1, \ldots, N$. Each row of the state matrix is a vector that characterizes each firm at time $t$, while each column represents the distribution of the corresponding variable at time $t$. Computations are performed on $R$ software.

In our simulations, we tend to choose total capital $M = 100$, normal profit rate $r = 0.2$, and number of firms $N = 10$. Initially, the cost structure is set to be homogeneous among firms, defining $ac_i = 5 \quad \forall i \in N$. Initial prices for each firm are obtained from a normal distribution around the price of production $p^* = ac(1 + r)$. Initial capital shares are obtained from a normal distribution around $1/N$ that we later normalize so that $\sum_i x_i = 1$. We show one simulation for $N = 10$ and 1000 timesteps in figure 1 and between $t = 150$ and $t = 200$ in figure 2. We also plot the change in inventory (that is, output produced minus output sold, i.e. $X - S$) with respect to the deviation of prices from the average

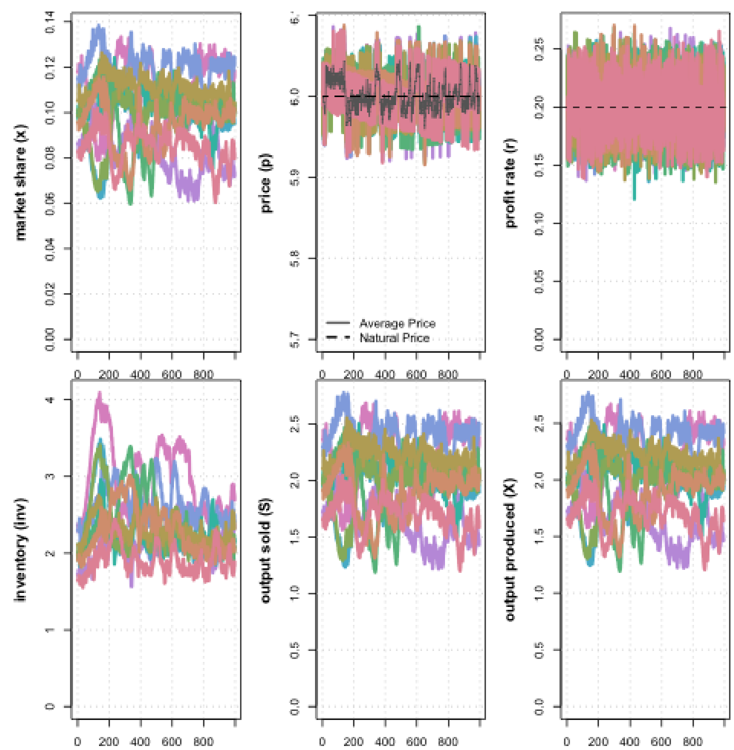


Figure 1: **Simulation of 1000 timesteps without technical change** Each colored line represents the trajectory of the corresponding microeconomic variable for one particular firm.

[figure 3]: we imposed market demand to flow from higher to lower prices, so we expect a strong positive correlation. Most importantly, the stochastic decentralized mechanism of price formation shows remarkable stability: prices set randomly by firms always gravitate around the price of production, even if we initialize the simulation with all prices set by firms either below or above the price of production $p^* = ac(1+r)$. We can find the stability of the mechanism in the double-exponential distribution of $\frac{1+r_i}{1+r}$, which are the eigenvalues of the dynamical system in the time-rule equation (6), which can be characterized as a Laplace distribution centered around 1 and with scale parameter 0.025. The distribution of growth rates also follow the double-exponential form 5, in agreement with the empirical evidence [Coad, 2009]. The price of production acts as a fixed point to the dynamic system, around which the average industry price gravitates under stochastic oscillations.

We can perform a particular simulation run of the algorithm many times in order to check its robustness. This allows to explore further how our competition mechanism induces stable, stationary distributions on the microeconomic variables [figure 4]. The time evolution of the capital share $x(t)$ takes the form

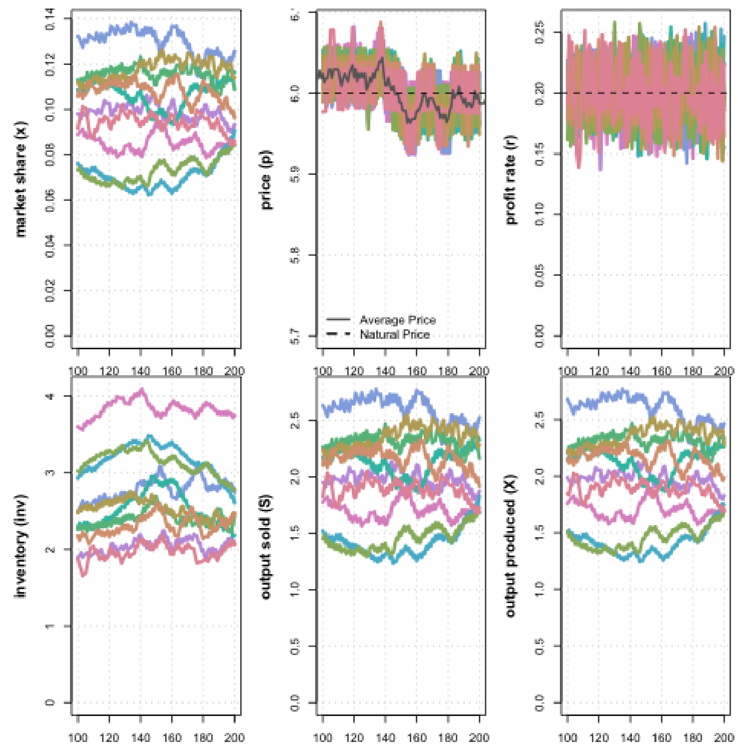


Figure 2: **The same simulation of 1000 timesteps without technical change only between** $t = 100$ **and** $t = 200$

of a random walk following the iteration rule in equation (6) around a mean $\langle x \rangle = 0.1 = 1/N$. Equation (6) shows a stochastic process regulated by profit rate $r$, which appears to feature (for the cubic drift term) a very clear fat-tailed double-exponential distribution centered on $\langle r \rangle = 0.2 = r$, i.e. the normal profit rate. Prices set by firms (with identical costs $ac = 5$) gravitate around the price of production, $p^* = (1 + r)ac = 6$. Inventory, output sold and output produced also show a stable pattern, all averaging at their corresponding share of the output, $\frac{M}{Nac} = 2$.

## 4.1 Introducing technical change

Now evolving, heterogeneous costs for firms can be explored. Firms with lower costs are able to enjoy greater profit rates thanks to their price-setting and thus eventually survive at the expense of their higher-cost competitors, which will die out. According to the classical theory of real competition, price equalization on firms with heterogenous costs induces a spectrum of profit rates among them. Our simulations offer many ways to explore the emergence of profit rate differentials due to differing production costs. For instance, technical change can be

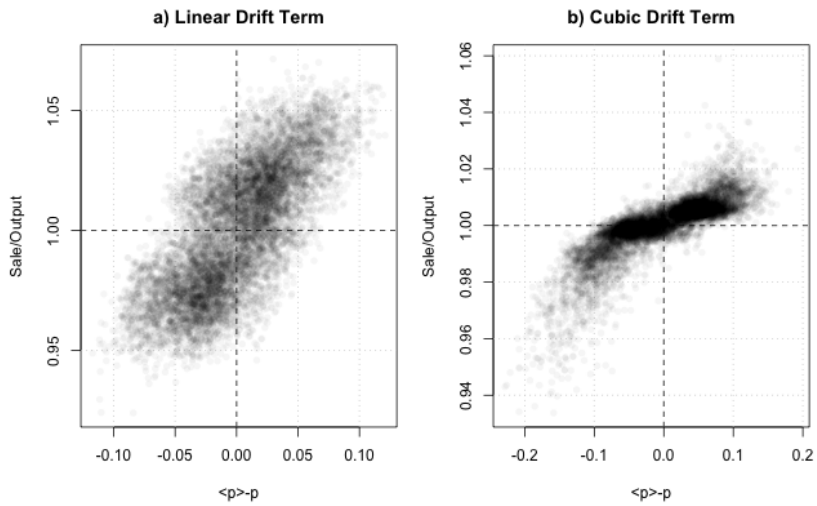


Figure 3: **Relation between the price deviation from the average and the sale/output ratio $S/X$ for two simulation runs using a linear drift term (a) and a cubic drift term (b)** This is the dynamical equivalent to the downward-sloping demand curve. An increasing relation is expected, so that a sale/output ratio above (below) unity corresponds to a positive (negative) price deviation $\langle p \rangle - p$. The dispersion away from strict linearity or the cubic S-shape is induced by the scale factor $\mu$.

exogenously introduced by lowering the costs of some of the firms at a particular timestep for one-off technical change or at a probability for persistent technical change. Figures 7 and 8 shows a single simulation where a portion of the firms lower their costs 0.1 units to $ac = 4.9$, eventually lowering the industry price to $p^* = 5.88$ and inducing profit rate differentials. In figure 8 the 'weighted price' is also depicted, computed as

$$p_w = \frac{M'}{X} = \frac{M(1+r)}{\sum_{i=1}^{N} \frac{M_i}{ac_i}} = \frac{M(1+r)}{\sum_{i=1}^{N} \frac{x_i M}{ac_i}} = \frac{1+r}{\sum_{i=1}^{N} \frac{x_i}{ac_i}} \tag{17}$$

The industry average price happens to gravitate around this value, which eventually becomes the new price of production when the capital shares of the higher-cost firms go to zero as they die out. $p_w$ becomes $(1+r)ac$ when costs are homogeneous among firms.

Figure 9 looks at the linear relationship that emerges between profit rate and the cost inverse by simulating $N = 200$ firms with total capital $M = 1000$ when costs are obtained from a uniform distribution between 5 and 5.5. The computational model provides a simple mechanism based on the classical theory of real competition that is also able to reproduce profit rate differentials, depending on the heterogeneity of costs across firms. The issue of differences in profit rates across firms and industries is "a, perhaps the, fundamental issue

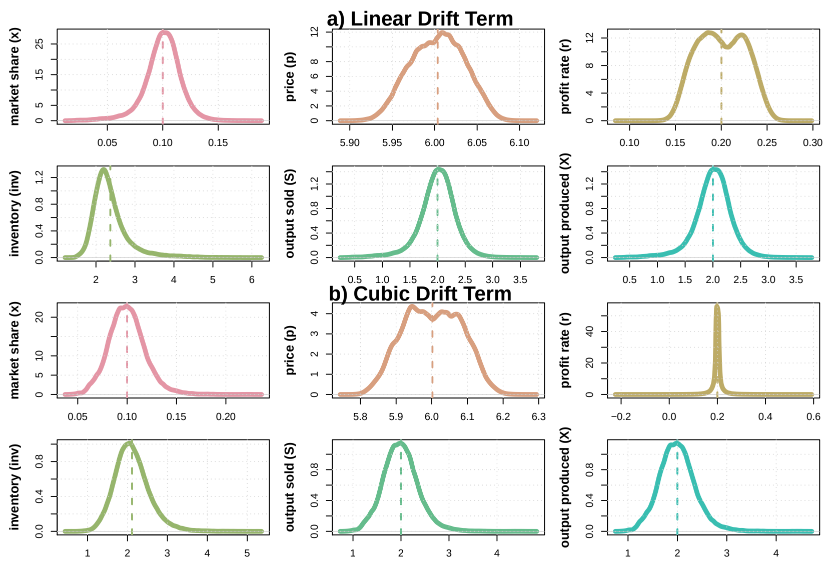


Figure 4: **Distribution of microeconomic variables for sets of many simulation runs** with no technical change for a) 250 simulations, 500 timesteps, $N = 10$ firms, and a linear drift term at (11) and for b) 100 simulations, 500 timesteps, $N = 10$ firms, and a cubic drift term.

in industrial organization" [Megna and Mueller, 1991, p.632]. What is shown with these simulations is that the requirement for firm survival is not profit maximization, but only a minimum of normal profits that ensure reproduction over time (or nonnegative in the neoclassical framework where normal profit is considered a capital cost).

# 5 Conclusions

This paper documents the simulations of a computational model of classical-evolutionary dynamics of price formation within a single industry under technical change, following the real-competition theoretical framework of Anwar Shaikh in *Capitalism* [Shaikh, 2016]. In classical political economy, competitive firms are viewed as profit-driven, price-setting and cost-cutting. In this paper, we have focused on the self-expansion of autonomous individual capitals, but their circuits are in fact crucially interlinked. They transform inputs into outputs and since some of their outputs are the production inputs of others, the whole set of interdependent processes of production self-reproduces as a whole albeit irregularly and turbulently, under constant creative destruction. Paraphrasing Piero Sraffa [1960], capitals self-reproduce themselves over time by producing commodities by means of commodities and labor (summarized as average costs): hence $M - C - P - C' - M'$ instead of $C - C'$ as in Sraffa.

In this sense, classical political economy closely resembles evolutionary the-

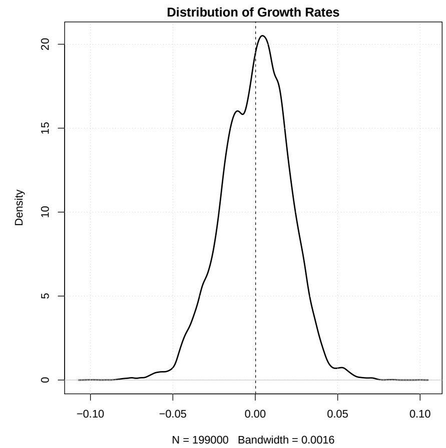


Figure 5: **Distribution of individual growth rates for a particular simulation** which follows the double-exponential shape of the empirical evidence [Coad, 2009]. As expected, the average growth rate in simple reproduction is zero.

ory on the origins of life as a self-reproducing set of interdependent chemical processes (of the form $A + B \rightarrow C$), with the crucial distinction that in political economy the interdependence of production processes is mediated by the price mechanism in monetary terms, rather than directly with only quantity concentrations. From an evolutionary viewpoint, intra-industrial real competition presents a simple yet powerful dynamical model of evolutionary selection. This is the underlying reason of our focus on the dynamics of decentralized price formation among firms within a single industry. Significantly enough, it is classical political economy and not evolutionary biology that provides a well-defined mechanism of evolutionary selection within a single industry: the evolutionary fitness of a process of production is defined by its profitability, while the notion in biology features the same problems of circularity than the neoclassical concept of utility and cannot be empirically measured as in economics.

Profitability, a measure of the evolutionary fitness that is well-defined with respect to microeconomic variables, regulates the stochastic growth of firms via trial-and-error price-setting. The stochastic process of competition still manages to induces stable, stationary distributions in the microeconomic variables that can be conceived as a 'statistical equilibrium' as Steindl [1965], Farjoun and Machover [1983], Mirowski [1989] or in Scharfenaker and Foley [2017]. In this case, the constraint at stake is of classical simple reproduction, the invariance of the industrial mass of revenue. The computational model consists of a simple recursive algorithm based on discrete iterations over a state vector. Critically, we did not assume the conventional notion of equilibrium as a matter of fact from which to derive logical results – an assumption that is often unwarranted and tends to obfuscate the actual economic dynamics at play [Mirowski, 1991; Pangallo et al., 2018]. We used the toy model to explore how standard assump-

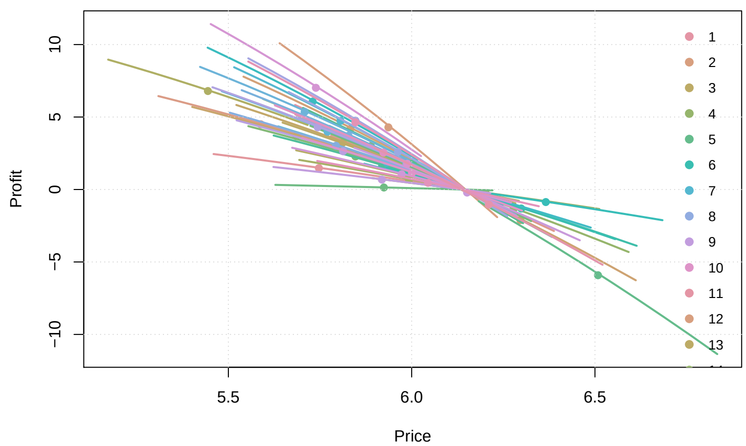


Figure 6: **Static price decisions of firms and their impact on own profits at a particular timestep** *ceteris paribus* all firms gain profits if they reduce their price with respect to the others, but interestingly enough we still retrieve something akin the famous kinked demand curve of Hall and Hitch. The intersection point is the price average, the dots represent the actual prices set by firms and the straight lines the potential profit they can obtain by changing their price keeping the others' fixed.

tions such as profit maximization or equilibrium may be actually irrelevant for economic analysis.

In sum, we have shown that the classical framework of real competition provides a parsimonious, yet powerful mechanism of the turbulent formation of a price of production without resort to any notion of optimization considerations but minimal rationality. The average industry price gravitates around a price of production as an attractor of stable oscillations. As well, our simple model can induce persistent profit rate differentials within a single industry, without any consideration of market power, exclusively based on the heterogeneous cost structure of firms.

Further research in this direction would aim at characterizing competition through product differentiation, introducing fixed capital and increasing returns to scale, addressing turbulent profit rate equalization in multi-sectorial self-reproduction in the form of "classical general equilibrium", and providing a mathematical formalization of the dynamic interdependencies underlying the stochastic growth of firms.

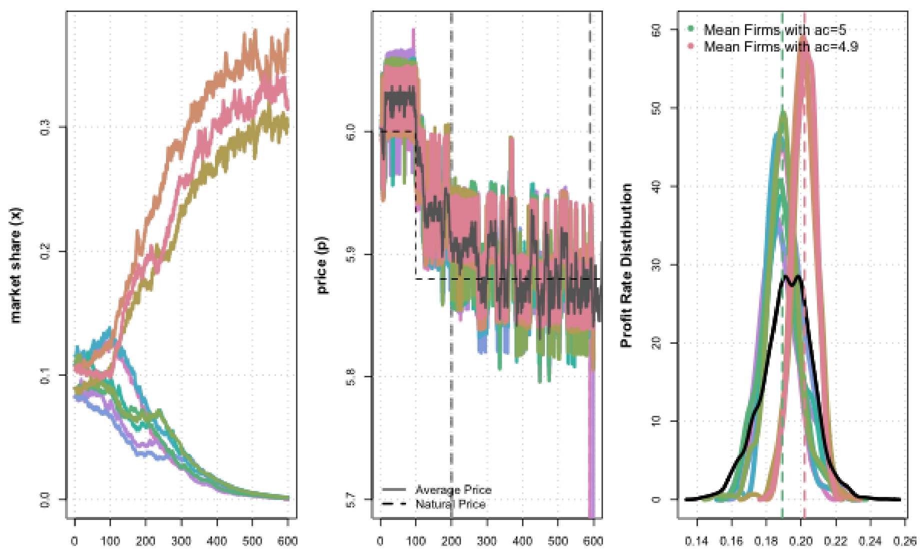


Figure 7: **Simulation from $t = 1$ to $t = 600$ of $N = 10$ firms with one-off technical change for 3 firms at $t = 100$** showing persistent profit rate differentials in the right panel between $t = 200$ and $t = 589$ when the price stabilizes around its new price of production (indicated by the vertical dashed lines in the center panel).

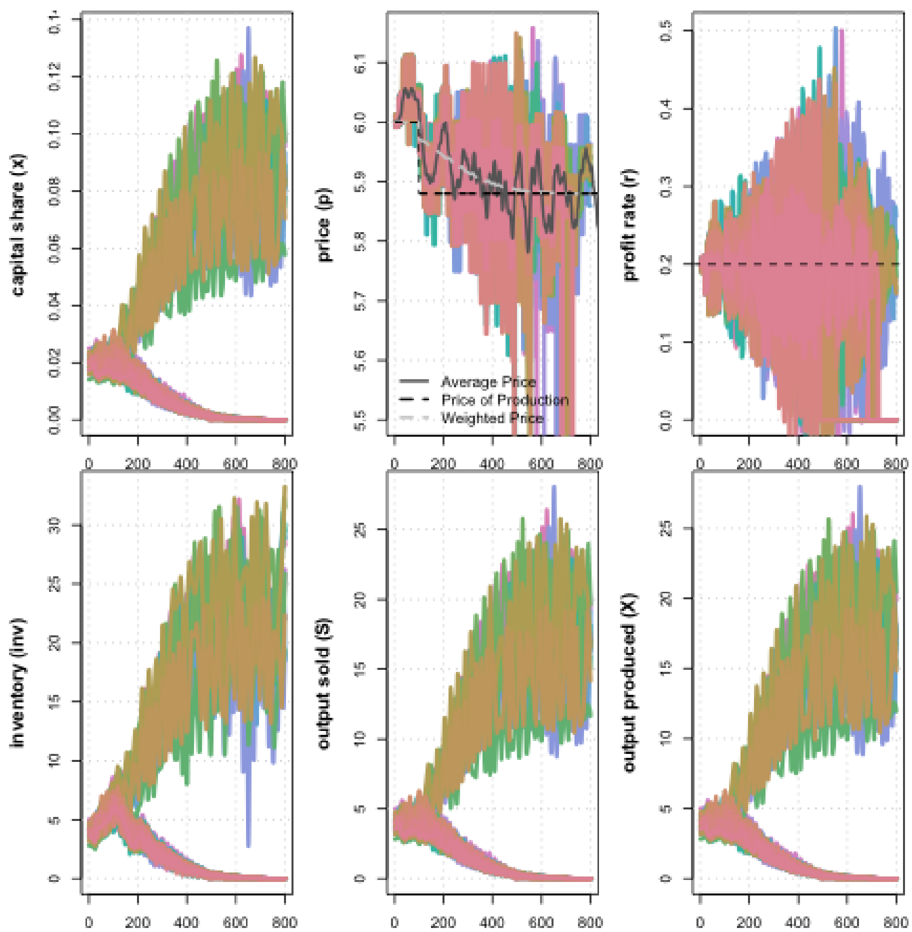


Figure 8: **Simulation from** $t = 1$ **to** $t = 800$ **of** $N = 50$ **firms with one-off technical change for 12 firms at** $t = 100$ with different specification for pricing rules. Average price gravitates around the weighted price.

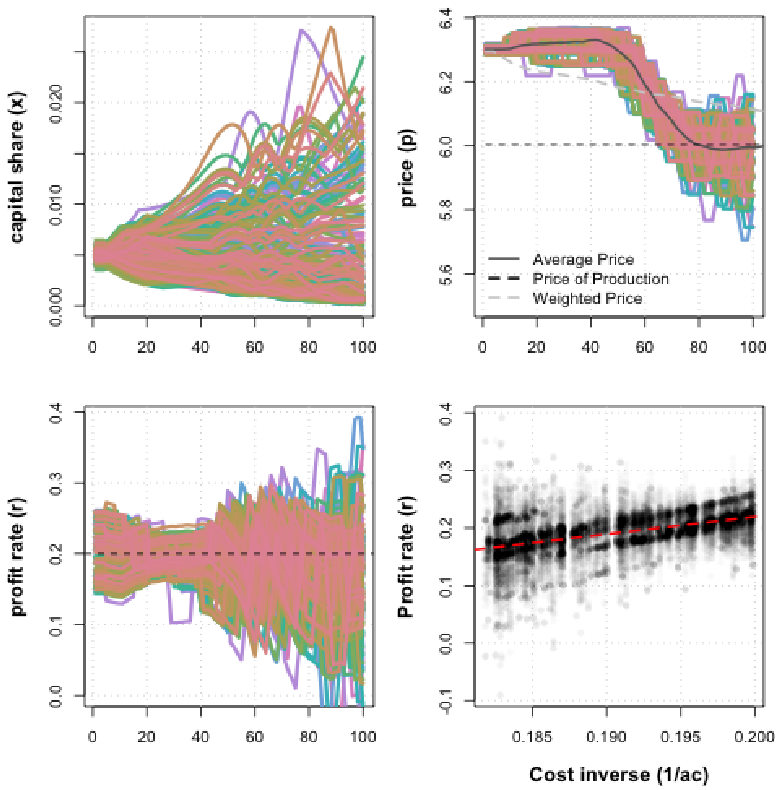


Figure 9: **Simulation from $t = 1$ to $t = 100$ for $N = 200$ firms with costs obtained from a uniform distribution between 5 and 5.5 units** In the bottom-right panel, one observes a linear relation between the profit rate and the inverse of costs.

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