The Conservation of Value Reconsidered: Hamiltonian Dynamics, the Transformation Problem, and the International Dissipation of Labour-Time

Oriol Vallès Codina

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Philip Mirowski argued that classical and neoclassical political economy share a founding ambition: to identify a conserved invariant playing the role of energy in physics. This paper argues that the classical research programme, when reconstructed through four complementary frameworks—Foley and Duménil's Single-System Labour Theory of Value (SS-LTV), the Hamiltonian formalism of Goodwin and Flaschel–Semmler, Jain and Krishna's autocatalytic set theory, and the Cambridge capital controversies—provides the only coherent economic realisation of that ambition. Section by section, the paper demonstrates: (i) how the SS-LTV dissolves the transformation problem by treating value and price as two expressions of the same system mediated by the Monetary Equivalent of Labour Time (MELT), preserving conservation at the aggregate level; (ii) how the Cambridge controversies—reswitching, capital-reversing, and Wicksell effects—destroyed the neoclassical claim to an independent measure of capital, vindicating the classical approach; (iii) how the Flaschel–Semmler Hamiltonian formalises value conservation in multi-sector reproduction, with Goodwin cycles as a special conservative case and crises as dissipative episodes; (iv) how Sraffa's economy is formally an autocatalytic set (ACS) in the sense of Jain and Krishna (1998): basic commodities are the dominant ACS, the standard commodity is its leading eigenvector, the maximum rate of profit measures the ACS's surplus catalytic productivity, and crises are phase transitions triggered by keystone-sector disruption; (v) how Foley and Smith's thermodynamic work at SFI identifies entropy and irreversibility—not the utility–energy metaphor—as the correct physics analogy; (vi) how Bukharin's disproportionality theory and Luxemburg's underconsumption argument reread as non-conservative departures from Hamiltonian conservation and as ACS disruptions respectively; (vii) how at the international scale, conservation breaks down into systematic value transfers from periphery to core due to differential organic compositions of capital (Işıkara and Mokre) and knowledge/complexity rents (Rotta); and (viii) how ecological scarcity introduces a permanent dissipative force in the form of Ricardian ground rent. The paper concludes that a unified theory of social reproduction—autocatalytic at its structural base, Hamiltonian in its conservative dynamics, thermodynamic in its irreversible production, and dissipative at its international and ecological margins—represents the most rigorous available fulfilment of the nineteenth-century project of economic invariants.

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1 Introduction

Philip Mirowski's *More Heat than Light* (1989) posed a challenge that remains unanswered at the centre of political economy: can economics possess a genuine conservation law, analogous to the principle of energy conservation in physics? Mirowski traced the nineteenth-century origins of both the classical and neoclassical research programmes to this shared aspiration. His verdict was damning on both counts—classical political economy left the invariant (labour-time) underspecified and mathematically underarticulated, while neoclassical economics imported the energy metaphor without the substance, producing a formalism that conserved only accounting identities.

This paper contests that verdict—but only for the classical tradition. The neoclassical programme's failure, as Mirowski documented, was real and deep: it is confirmed by the Sonnenschein–Mantel–Debreu (SMD) theorem, the integrability critique articulated by Wade Hands (1993), and—most decisively—by the Cambridge capital controversies whose full implications Mirowski did not systematically pursue. But the classical tradition, when reconstructed through the contributions of four decades of heterodox economics, can be shown to provide a rigorous, empirically grounded, and mathematically coherent conservation law for economic systems.

The argument proceeds in three movements. The first reconstructs the conservation of value at the level of the closed capitalist economy: we show that Foley and Duménil's Single-System Labour Theory of Value (SS-LTV) resolves the transformation problem not by abandoning conservation but by identifying its correct level—the net product, mediated by the MELT—and that the Hamiltonian framework of Goodwin and Flaschel–Semmler provides the dynamical expression of this conservation in multi-sector reproduction. The second movement extends the analysis to international exchange: drawing on Işıkara and Mokre's (2021) empirical price-value deviation analysis and their theoretical elaboration in *Marx's Value Theory at* *the Frontiers* (2026), alongside Rotta and Kumar's (2023) work on productive/unproductive classification and complexity rents, we show that conservation systematically breaks down at the global level in predictable ways, generating value transfers from the periphery to the core. The third movement addresses what Foley (Understanding Capital, 1986) called the *specific theories of crisis*—disproportionality, underconsumption, and the falling rate of profit—and reinterprets them as distinct modes of dissipation in the Hamiltonian system, connecting Bukharin and Luxemburg to the dynamics of expanded reproduction. A coda addresses ecological ground rent as a fundamental and growing dissipative force.

The paper aims to be a contribution to the *Review of Political Economy* readership: it assumes familiarity with Sraffian and post-Sraffian debates, with the post-Keynesian tradition, and with the Marxian value-theoretic controversies, while seeking to show that a unified framework is not only possible but necessary for addressing contemporary challenges in heterodox political economy.

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2 Mirowski's Challenge: The Two Research Programmes and Their Failure

Mirowski's central historical claim is that the so-called marginal revolution of the 1870s—Jevons, Walras, Menger—was not the spontaneous discovery of rational choice theory but a systematic transplantation of classical mechanics into economics. The target was to achieve in economics what Helmholtz, Joule, and Clausius had achieved in physics: a principle of invariance that would allow the system's transformations to be measured against a constant background.

The neoclassical attempt located the invariant in utility. Walras explicitly modelled exchange as force balance—marginal utility as generalized force, quantities as displacements, budget constraints as energy surfaces. The fundamental differential relation $dU = \sum_i p_i dx_i$ implied a potential function analogous to mechanical energy, with demand as its gradient. Slutsky symmetry guaranteed that compensated substitution effects formed a conservative vector field; Afriat's Theorem (1967) later gave this an empirical interpretation through the Generalized Axiom of Revealed Preference.

But as Hands (1993) clarified, this conservation lives only in *compensated* (Hicksian) demand space, not in observable Walrasian demand. The conserved object is the expenditure function, not utility: the analogy is tautological at its core, resting on the accounting identity that income equals expenditure. The Friedman (1953) *as if* defence institutionalised the abandonment of ontological grounding: models became predictive devices, not representations of causal invariants. And the SMD theorem (Sonnenschein 1972; Mantel 1974; Debreu 1974) completed the demolition at the aggregate level: even well-behaved individual preferences yield no stable aggregate demand structure, annihilating any claim to a macro-level conservation law.

The classical programme located the invariant in labour-time. Exchange conserves value; production—through the application of living labour—creates new value. This appears to provide a genuine energy analogue: labour as source term, exchange as conservative manifold. But Mirowski complained that the classical tradition, including Marx, left this invariant mathematically underspecified: the transformation problem (how values become prices) appeared to violate conservation, and the relationship between the labour-value system and the price system remained contested for a century.

The reconstruction we offer suggests that Mirowski was right about the failure of specific formulations, but wrong about the failure of the programme. The transformation problem has been dissolved—not solved—by the SS-LTV, and the Hamiltonian dynamics of reproduction provide precisely the mathematical articulation that Mirowski found lacking.

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3 The Transformation Problem and the SS-LTV: Conservation at the Right Level

The transformation problem arises from what Duménil and Foley (2008) call the *dual-system* approach inherited from Ricardo and adopted, ambiguously, by Marx. In the dual-system framework, values (labour-times embodied in commodities) and prices of production (prices equalizing profit rates across sectors) are two separate systems, and the problem is to show they are consistent. The alleged contradiction is that Marx's two invariance conditions—sum of values equals sum of prices; sum of surplus-values equals sum of profits—cannot both hold simultaneously when inputs are revalued at prices of production.

The SS-LTV, proposed independently by Duménil (1980, 1983) and Foley (1982), resolves this by abandoning the dual-system premise. There is only one system of prices in a capitalist economy, and value categories are defined within it. The key move is the definition of the *Monetary Equivalent of Labour Time* (MELT):

$$\mu = \frac{p^\top (I - A)x}{\ell^\top x}$$

where $p^\top(I-A)x$ is the price of the net product (national income in monetary terms) and $\ell^\top x$ is the total living labour expended. The MELT is the conversion factor between labour-time and money: it expresses quantitatively what Marx called the *price form of value*.

Under this definition, the first Marxian invariance condition holds as an identity: the price of the net product equals the monetary expression of total living labour. Conservation is secured not at the level of individual commodities (where prices of production deviate from direct prices) but at the aggregate level of the net product. The deviation of individual prices from values is a redistribution of surplus-value among sectors through competition, not a violation of conservation: it is, to use Foley's analogy from *Understanding Capital* (1986, p. 6), like the redistribution of value added between capitalists and workers—the conservation principle operates at the aggregate level and shows itself in the average behaviour of the system.

This is not a mere accounting trick. As Foley (1986, p. 6) stresses, "the labour theory of value appears at the level of the whole system of commodity production to be such a conservation law—value is produced by labor and conserved in exchange." The MELT is an empirically observable quantity—the ratio of money GDP to total hours of productive labour—and it fluctuates with productivity changes, technology shocks, and distributional shifts. It is, in the language of Mirowski's analogy, the economic equivalent of the conversion factor between joules and calories: not a constant of nature, but a measurable and theoretically grounded conversion ratio.

The competing Single-System interpretations differ in how they treat the value of labour-power: the TSS (Temporal Single-System) approach of Freeman and Carchedi values constant capital at historical cost, making the MELT time-path dependent; the SS-LTV values it at current (replacement) cost, consistent with Marx's own statement that "the value of a commodity is not determined by the labour-time originally expended in its production, but by the labour-time required for its reproduction" (*Capital* III.24). The replacement-cost approach is more consistent with the Sraffian long-period method: values and prices are simultaneously determined, and the MELT provides a bridge between the two systems of accounts.

The Sraffian input-output equation $p^\top x = p^\top Ax + w\ell^\top x$, which Sraffa (1960) wrote as the basic accounting identity of production, is in this light not a *refutation* of the LTV—as the neo-Ricardian school (Steedman 1977; Garegnani 1984) argued—but its aggregate expression. The net product $p^\top (I - A)x = w\ell^\top x + \Pi$ decomposes into wages and profits; the MELT normalisation $\mu = p^\top (I - A)x/\ell^\top x$ makes explicit that the wage-profit division is a division of the labour newly added, not a determination of prices from outside the value system. Shaikh's (1984) gauge-condition interpretation of Sraffian normalisation is fully consistent with this: the choice of normalisation is a choice of the reference level of the value potential, not a substantive constraint.

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4 The Cambridge Controversies: Capital Measurement and the Vindication of Classical Economics

The Cambridge capital controversies of the 1950s–1970s, surveyed definitively by Cohen and Harcourt (2003), provide the most devastating critique of the neoclassical programme from within mathematical economics itself—and, simultaneously, the strongest external vindication of the classical approach.

The neoclassical theory of distribution rests on three "parables" (Samuelson 1962): (1) the real return on capital is determined by the marginal product of a physical quantity of capital; (2) a greater quantity of capital is associated with a lower rate of interest; (3) income distribution between labour and capital is explained by factor scarcity. All three require that capital can be measured in physical units independent of distribution.

The problem, identified by Robinson (1953–54) and formalised by Sraffa (1960), is that capital goods are heterogeneous commodities whose relative values depend on the rate of profit. The measurement of "capital" as an aggregate therefore presupposes the rate of profit, which it is supposed to determine. This circularity—the Wicksell effect—yields the phenomena of *reswitching* (the same physical technique being optimal at two different profit rates with an alternative preferred between them) and *capital-reversing* (a lower rate of interest associated with a lower capital-labour ratio). Both phenomena invalidate parables 2 and 3; by Samuelson's (1966) own admission, the anomalies are "not… an empirical fluke but a *logical* possibility."

What does this establish? Negatively, it destroys the neoclassical claim that distribution is determined by marginal products of independently measured factor quantities. Positively—and this is the connection that Mirowski did not emphasise—it vindicates the classical and Sraffian approach in which distribution is determined at a prior level: the wage-profit frontier, derived directly from the input-output structure and the MELT, without recourse to any aggregate quantity of capital.

The Sraffian standard commodity (Sraffa 1960; Eatwell 1975) provides the clearest illustration. By constructing an industry with the same input-output proportions as the whole economy, Sraffa identified a composite commodity in which the wage-profit trade-off is linear: $r = R(1 - w/w_{\max})$, where R is the maximum rate of profit. This is not a metaphor but a precise algebraic result: the standard commodity *is* the invariant measure of surplus, immune to price changes arising from distribution shifts. It is Sraffa's realisation of Ricardo's "invariable measure of value"—the object Ricardo sought but could not construct.

The Cambridge controversies thus demonstrate two things simultaneously: (i) that the neoclassical production function aggregates capital incoherently, yielding no genuine invariant; and (ii) that the classical tradition *can* construct an invariant measure of surplus (the standard commodity, the MELT) that is independent of distribution in precisely the way Ricardo sought. Mirowski's complaint that classical economics left its invariant underspecified is met, in retrospect, by Sraffa's construction.

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5 The Hamiltonian Structure of Reproduction

5.1 Goodwin's Classical Growth Cycle

The simplest Hamiltonian model of value circulation is Goodwin's (1967) predator-prey system coupling employment λ and wage share ω:

$$\dot{\lambda} = \lambda(a - b\omega), \qquad \dot{\omega} = \omega(-c + d\lambda).$$

Defining $q = \ln \lambda$ and $p = \ln \omega$, the system becomes:

$$\dot{q} = a - be^p, \qquad \dot{p} = -c + de^q,$$

which is generated by the Hamiltonian:

$$H(q, p) = ap - be^p + cq - de^q, \qquad \dot{H} = 0.$$

The conservation $\dot{H} = 0$ expresses that the redistribution of value between wages and profits—within the circuit of capital—does not create or destroy value. The closed orbits of the Goodwin system represent oscillations in the wage-profit division that are conservative at the level of the total value of the net product: a rising wage share depresses the profit rate, slowing accumulation and employment, which then restores profitability in a perpetual cycle. No net value is created or destroyed in this redistribution; the Hamiltonian level—the total value of the net product, denominated in MELT units—is constant on each orbit.

Departures from conservation arise when fiscal policy, credit expansion, or technical change introduce exogenous forces. Mathematically, adding a perturbation $\epsilon F(q, p)$ to the Goodwin system yields $\dot{H} = \epsilon\{F, H\}_{\text{Poisson}} \neq 0$: the Hamiltonian level drifts, either upward (value accumulation, expanded reproduction) or downward (crisis, devaluation). This provides a precise criterion for distinguishing redistributive from generative or destructive dynamics: redistributive dynamics preserve H; creative/destructive dynamics shift it.

5.2 The Flaschel–Semmler Multi-Sector System

Flaschel and Semmler (1987) extend this structure to an n-sector economy. Let $M = B - RA$ encode excess-demand and excess-profit couplings between sectors, where B is the output matrix, A the input matrix, and R a uniform profit factor. Price and quantity adjustments follow:

$$\dot{y} = -\Delta_p M e^z, \qquad \dot{z} = \Delta_x M^\top e^y,$$

where $y = \ln p$, $z = \ln x$. When the adjustment speed matrices are proportional to the identity ($\Delta_p = \delta_p I$, $\Delta_x = \delta_x I$), the system is generated by the Hamiltonian:

$$H(z, y) = \delta_x \mathbf{1}^\top M^\top e^y + \delta_p \mathbf{1}^\top M e^z, \qquad \dot{H} = 0.$$

This Hamiltonian has a precise economic interpretation. The first term, $\delta_x \mathbf{1}^\top M^\top e^y$, captures the adjustment of quantities driven by price signals—the gravitational pull of profit differentials on sectoral outputs. The second term, $\delta_p \mathbf{1}^\top M e^z$, captures the adjustment of prices driven by quantity imbalances—excess demand and supply pressures on prices. Together, they describe a conservative field of intersectoral value flows: value redistributed among sectors through competition, without net creation or destruction.

When adjustment speeds differ across sectors ($\Delta_p \neq \delta_p I$), the system becomes non-conservative:

$$\dot{H} = \sum_i \Pi_i^{(x)} - \sum_i \Pi_i^{(p)},$$

where $\Pi_i^{(x)} = \delta_x \delta_p x_i [(M^\top p)_i]^2$ and $\Pi_i^{(p)} = \delta_x \delta_p p_i [(Mx)_i]^2$ measure sectoral power flows—rates of value creation or dissipation by structural asymmetries. Financially driven distortions, supply chain bottlenecks, and monopoly pricing all enter as asymmetric adjustment speeds that drive $\dot{H} \neq 0$.

The equilibrium of the Flaschel–Semmler system corresponds to the Sraffian long-period position: prices proportional to production prices, quantities in the proportions of the standard commodity. This is the "gravitational centre" of the value field—the state toward which the conservative dynamics tend in the absence of perturbations. Numerically, simulation of Sraffa's three-commodity example (wheat, iron, pigs) using symplectic integration confirms that H remains constant under simple reproduction, with relative shares oscillating around Sraffian proportions, while a growth rate $g > 0$ (expanded reproduction) yields a monotonically increasing H.

5.3 MELT, Energy Decomposition, and the Value Potential

The MELT $\mu = V/L$—total value over total labour—provides the link between the Hamiltonian and measurable quantities. Decomposing the Hamiltonian:

$$H = K + T, \quad K = p^\top \Delta_p M e^z, \quad T = x^\top \Delta_x M^\top e^y.$$

The *kinetic* term K represents value in motion—the adjustment pressure of prices on quantities (the profit-rate signal propagating through the economy). The *potential* term T represents stored value—the adjustment pressure of quantities on prices (excess demand lodged in sectoral disequilibria). In simple reproduction, $\dot{H} = 0$: kinetic and potential value exchange continuously, as in a frictionless pendulum. The MELT μ converts the Hamiltonian level H to labour-time units: H/μ is the total labour-time embodied in the system's disequilibrium—a measure of how far the economy deviates from its Sraffian equilibrium in labour-value terms.

6 The Sraffian Economy as an Autocatalytic Set

6.1 Catalytic Closure and the Structure of Production

Sraffa's system of production is not merely a set of simultaneous equations: it is, in the precise sense introduced by Jain and Krishna (1998), an **autocatalytic set** (ACS). Understanding this identity—rather than treating it as a loose analogy—clarifies why conservation of value is not an imposed theoretical assumption but an emergent property of any sufficiently dense system of interdependent production.

Jain and Krishna define an ACS as a subgraph of a directed interaction network in which *every node has at least one incoming catalytic link from within the same subgraph.* This is the property of **catalytic closure**: the set collectively provides the catalysts for its own reproduction. No external input is required to sustain the set's activity; it is self-catalysing. In chemical systems, the simplest ACS is a two-cycle in which molecule A catalyses the production of B and B catalyses the production of A. In more complex ACSs, the closure may involve chains of catalytic support spanning many species.

The application of autocatalytic set theory to economics is not without precedent. Hordijk et al. (2022) have combined the TAP model of combinatorial technological evolution with RAF (Reflexively Autocatalytic and Food-generated) set theory, demonstrating that product-transformation networks arising from combinatorial innovation have a high probability of containing autocatalytic sets and arguing that the economy can be viewed as an autocatalytic set — a result they call "economic autocatalytic sets." Separately, Emmenegger et al. (2020) have established the mathematical correspondence between Sraffa's and Leontief's input-output systems through the Perron-Frobenius theorem, demonstrating the central organising role of the dominant eigenvalue and eigenvector in both frameworks. What has not been done in the existing literature is to combine these two threads: to identify Sraffa's *basic commodities* with the dominant ACS in Jain and Krishna's dynamical formulation, and to show that the *standard commodity* is simultaneously the leading eigenvector of the production matrix and the attractor of the ACS self-reproductive dynamics. This identification — from catalytic closure to the conservation of value — is the specific contribution of the present section.

Sraffa's economy satisfies this definition exactly. Let the $n \times n$ input-output matrix A be the adjacency matrix of the production network, where $A_{ij} > 0$ if sector j's output enters as an input to sector i. Sraffa's **basic commodities**—those that "enter directly or indirectly into the production of all commodities" (Sraffa 1960, p. 7)—form precisely the dominant ACS of this network. A basic commodity is one that lies on a path in the production graph connecting every other commodity to every other: its removal would disconnect the network. The non-basics are the "leaves" of the graph—they receive catalytic support from the basics (they require basic inputs to be produced) but do not return catalytic support to all others (their outputs are not universally required as inputs).

This identification is formally tight. Jain and Krishna prove, using the Perron-Frobenius theorem, that for a non-negative adjacency matrix C representing the catalytic interaction graph, the asymptotically stable attractor of the population dynamics

$$\dot{x}_i = \sum_j c_{ij} x_j - x_i \sum_{k,j} c_{kj} x_j$$

is the leading eigenvector $\mathbf{x}^{\lambda_1}$ of C, corresponding to the eigenvalue λ_1 of largest real part. In Sraffa's system, the same theorem applies to the matrix A: the *standard commodity* is the leading eigenvector of A, providing the proportions of the "standard system" in which the wage-profit frontier is linear. The standard commodity is therefore not an algebraic curiosity invented for purposes of normalisation: it is the *attractor of the production network's own self-reproductive dynamics*, the equilibrium distribution to which competition drives the relative magnitudes of sectoral outputs.

6.2 Eigenvalue Conditions and the Rate of Profit

The Jain-Krishna framework illuminates the relationship between the production structure and the maximum rate of profit. In their model, $\lambda_1 = 0$ (no ACS) means populations decay to zero — the network cannot sustain itself. $\lambda_1 \geq 1$ means the ACS is growing: its aggregate catalytic throughput exceeds its natural decay rate.

In Sraffa's system, λ_1 is the dominant eigenvalue of A. Three cases:

- $\lambda_1 = 1$: The economy is at the boundary of viability — it exactly reproduces itself with no surplus. Sraffa calls this the *subsistence economy*. In JK terms, the ACS is self-sustaining but not growing.
- $\lambda_1 < 1$: The economy produces a surplus — more is produced than is consumed as input. The maximum rate of profit is $R = (1/\lambda_1) - 1 > 0$. In JK terms, the ACS has positive net catalytic throughput: it generates more "population" than it consumes.
- $\lambda_1 > 1$: The economy is sub-viable — inputs exceed outputs in value terms. No sustained production is possible. JK's equivalent is a network with insufficient catalytic density to maintain an ACS.

The maximum rate of profit R is therefore a direct measure of the ACS's net catalytic productivity: how much the self-reproducing network generates above its own reproduction requirements. The surplus is the excess above the "catalytic self-maintenance threshold." Distribution of this surplus—the wage-profit frontier $r = R(1 - w/w_{\max})$ in the standard system—is a question of how the excess catalytic product of the network is divided between the labour that performs catalysis and the capital that owns the network's nodes.

6.3 Keystone Sectors, Crisis, and ACS Disruption

The Jain-Krishna dynamics of graph evolution provide a precise mechanism for understanding economic crises that complements the Hamiltonian dissipation framework of Section 5. In JK, the "least fit" node—the species with smallest population under the current network—is selected for mutation. When this node happens to be a *keystone species* of the dominant ACS (a node with many outgoing catalytic links supporting other nodes), its mutation disconnects a large portion of the network from the ACS. Fig. 3 of Jain and Krishna (1998) shows the sharp drop in s_1 (the size of the dominant ACS) at $n = 4910$, when a dominant-ACS node mutates: the cascade of disconnections is sudden and severe.

In economic terms, a keystone sector is one whose outputs are universally required as inputs across the production network: energy, key intermediates, financial infrastructure, transport. When such a sector contracts sharply—whether due to supply shock, financial crisis, or deliberate restructuring—it severs catalytic links to many downstream sectors simultaneously. The result is not a smooth, conservative redistribution (the near-$\dot{H} = 0$ regime of Flaschel-Semmler near equilibrium) but a discontinuous collapse in the connectivity of the production network: an ACS disruption.

This provides a more fundamental account of crisis dynamics than the Hamiltonian approach alone. The Hamiltonian framework characterises crises as departures from conservation ($\dot{H} \neq 0$) but does not specify *why* such departures are sudden and cascading rather than smooth. The ACS framework answers this: crises are sudden because the production network has a *threshold property*—below a critical connectivity level, the dominant ACS dissolves, and the cascade of disconnections is self-reinforcing. This is the economic equivalent of a phase transition, not a smooth departure from equilibrium.

Bukharin's disproportionality theory acquires additional precision here: the "overexpanded department" in his crisis theory corresponds to a sub-network that has grown beyond the catalytic support provided by the rest of the production network. Its products are not sufficiently required as inputs elsewhere; it does not form a self-sustaining ACS with the rest of the economy. When profitability forces contraction, it is precisely the least-catalytically-integrated nodes that are shed first—and if any of these happen to be keystones for other sectors, the disproportionality crisis becomes a cascade.

6.4 Complexity Growth and the Evolution of the Production System

The most striking feature of Jain and Krishna's results is the dynamic: starting from a sparse random graph, the network *inevitably* evolves to an ACS, which then drives exponential growth in connectivity until it spans the whole graph. The time scale for ACS appearance scales as $\tau_a \sim s/m^2$ (where s is the number of species and m is the catalytic probability), and the growth time scale as $\tau_g \sim 1/p \sim s/m$.

In economic terms, this formalises the classical insight that capitalist production is self-expanding: the development of the division of labour, the emergence of new sectors, and the intensification of inter-industry linkages are not contingent historical accidents but the expected dynamic of an autocatalytic system subject to competitive selection. Technical change (the "mutation" of least-fit sectors) drives the evolution of the production network toward higher connectivity, greater specialisation, and deeper interdependence — precisely the "civilising mission" of capitalism that both Smith and Marx identified, and that Sraffa's system, read statically, tends to obscure.

The ACS framework therefore unifies the static (Sraffian) and dynamic (Marxian) dimensions of the classical tradition: Sraffa's simultaneous equations describe the *attractor* of the catalytic network (the standard commodity as leading eigenvector); Marx's analysis of technical change, competition, and crisis describes the *evolutionary dynamics* by which the network is driven toward and away from this attractor. The conservation of value in exchange is the property of the attractor state; the creation of value in production is the thermodynamic source term; and crisis is the phase transition that occurs when the network's ACS structure is disrupted beyond the threshold of catalytic closure.

Foley and Smith's Santa Fe Institute lectures (2007) mark a significant refinement of Mirowski's physics analogy. Where Mirowski focused on the mechanics–economics parallel (Hamiltonian, potential energy, force), Foley and Smith argued that *thermodynamics* provides a more accurate and productive framework than *classical mechanics* for economic analysis.

The Walrasian analogy—equilibrium as force balance, utility as potential energy—maps onto classical mechanics. But classical mechanics describes *reversible* processes: a Hamiltonian system run backward in time is indistinguishable from one run forward. Economic processes are not reversible: production consumes labour-time irreversibly; commodities are produced and then consumed; capital is accumulated and then depreciated. These features call for thermodynamics, not mechanics.

The relevant thermodynamic concept is entropy. Foley and Smith (2007) argue that the economic analogue of thermodynamic entropy is the *distribution* of value across agents and sectors—a measure of the disorder of the value field. In thermodynamic equilibrium, entropy is maximised subject to the conservation of total energy. In economic equilibrium (competitive long-period position), the "entropy" of value distribution is maximised subject to the conservation of total value: competition equalises profit rates, which is equivalent to maximising the number of ways a given total value can be distributed across sectors subject to the rate-of-profit constraint.

This interpretation has two important implications. First, it reconciles the conservative Hamiltonian dynamics (Goodwin, Flaschel–Semmler) with the irreversibility of production: the Hamiltonian describes the circulation of value *within* a period; the creation of new value by living labour is the thermodynamic source term that increases total "energy" from one period to the next. Second, it locates crises as thermodynamic in character: periods of rapid entropy decrease (forced synchronisation of economic activity, as in a financial crisis) followed by rapid entropy increase (liquidation, devaluation, restructuring). The business cycle is a sequence of entropy fluctuations around the competitive equilibrium.

Critically, Foley and Smith argue that the correct analogy between thermodynamics and economics is *not* through the neoclassical identification of utility with energy. Utility is not an entropy either: the SMD theorem shows that individual utility maximisation does not aggregate to a maximum-entropy distribution. The correct analogy runs through the distribution of productive activity and value across an economy characterised by irreversible labour expenditure—precisely the classical framework.

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7 Crises as Dissipation: Disproportionality, Bukharin, and Luxemburg

7.1 The Three Theories of Crisis

Foley's *Understanding Capital* (1986, Ch. 9) identifies three broad categories of Marxian crisis theory: disproportionality, underconsumption, and falling rate of profit. Each corresponds to a distinct mode of dissipation in the Hamiltonian framework.

7.2 Disproportionality: Non-Conservative Bifurcation

The disproportionality theory locates crisis in the mismatch between the two departments of production—Department I (means of production) and Department II (means of consumption). Marx's reproduction schemes require that the value of means of production purchased by Dept II exactly equals the value of consumer goods purchased by Dept I: $c_2 = v_1 + s_1$ in simple reproduction; for expanded reproduction, a specific growth-rate condition must hold.

Bukharin (in his critique of Luxemburg and in *The Economics of the Transition Period*) took this proportionality condition as the key: crisis arises when anarchic capitalist investment violates it, generating excess capacity in one department and deficiency in the other. The Marxist theorist of disproportionality, as Foley notes (1986, p. 148), argues that the contraction of the overexpanded department is not matched by expansion in the underinvested one; aggregate demand falls, and a crisis of realisation spreads across both departments.

In the Hamiltonian framework, this maps precisely onto the non-conservative case ($\dot{H} \neq 0$): asymmetric adjustment speeds between Department I and Department II sectors generate sectoral power flows $\Pi_i^{(x)} \neq \Pi_i^{(p)}$, and the Hamiltonian level changes. If Department I expands faster than the Sraffian proportions require, it accumulates excess productive capacity whose value cannot be realised: potential energy builds up in the T term without a corresponding increase in kinetic value K. The release of this pressure—devaluation of capital goods, bankruptcy, forced liquidation—is the dissipative episode of the crisis: a sudden decrease in H as stored potential value is destroyed.

The Flaschel–Semmler framework captures this formally through the profitability coupling matrix $M = B - RA$: if the growth rates of sectors deviate from those implied by the standard commodity proportions, the off-diagonal terms of M generate non-zero $\dot{H}$. The crisis threshold corresponds to a bifurcation in the dynamical system—the transition from stable oscillations (near-conservative) to divergent trajectories (strongly dissipative) as sectoral disproportions exceed a critical magnitude. This provides a rigorous mathematical realisation of Bukharin's intuition: it is not the level of accumulation but its sectoral disproportionality that generates the non-conservative dynamics leading to crisis.

7.3 Luxemburg's Underconsumption: External Demand as Entropy Sink

Rosa Luxemburg's argument in *The Accumulation of Capital* (1913) is structurally different from Bukharin's. She claims that capitalist economies are structurally incapable of realising the total surplus value produced in expanded reproduction without recourse to external (non-capitalist) demand. Foley (1986, pp. 150–151) reconstructs the argument: even if capitalists have the incomes to spend, it is unreasonable to suppose they will invest enough in expanding total capacity to maintain aggregate demand indefinitely—they will invest only as long as this is profitable, but the process of accumulation constantly raises the rate of surplus value and reduces the consumption base on which investment depends.

In the thermodynamic reading, Luxemburg's argument identifies a structural entropic gradient: the capitalist circuit $C - M - C'$ requires that money be advanced and returned with a surplus, but this surplus must be realised against a demand that grows at least as fast as supply. Internally, the capitalist class cannot provide this demand—their own consumption is constrained by the circuit's logic ($M - C - M'$: they buy in order to sell more). External demand—from non-capitalist producers, from colonies, from pre-capitalist household production—functions as an *entropy sink*: it absorbs the excess value produced, maintaining the thermodynamic gradient that drives the system.

This connects imperialism to the physics analogy: imperialism is the expansion of the capitalist system boundary in order to maintain the entropy gradient required for continued accumulation. The conquest of new markets is not primarily a moral failing but a *thermodynamic necessity* of the capitalist circuit—a system that can only be conservative at the aggregate level by continuously enlarging the aggregate. When external markets are exhausted (or when the periphery itself becomes capitalist), the entropy sink disappears and the system must either accept stagnation or find new sources of non-conservative expansion (credit money creation, public debt, financialisation).

Foley (1986, p. 151) is sceptical of Luxemburg's premise that investment must ultimately aim at workers' consumption—noting, correctly, that Marx's own formulations make capital accumulation the *telos* of capitalist production, not workers' consumption. But the thermodynamic reading rehabilitates her structural insight without relying on the teleological premise: it is sufficient that the circuit $C - M - C'$ requires a monetary return commensurate with the expanded value of C', and that this return cannot be sourced purely from within the circuit without requiring an ever-accelerating rate of internal credit creation—which is itself a form of non-conservative (dissipative) dynamic, as Foley's (1987) model of financial cycles demonstrates.

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8 International Value Transfers: Ișıkara–Mokre and Rotta

8.1 The Empirical Foundation: Turbulent Equalization

Işıkara and Mokre's (2021) empirical analysis across 42 countries using WIOD provides the direct empirical test of the value conservation claim. If value is conserved in exchange—if prices gravitate around values in the sense of the classical tradition—then price-value deviations should be bounded and systematic. Their results confirm this: deviations of market prices from direct prices (MP/DP) are 10–25%; deviations of prices of production from direct prices (PP/DP) are approximately 10%. The smaller PP/DP deviation confirms the classical gravitation mechanism: *prices of production are a closer attractor than market prices*, consistent with the turbulent equalization of profit rates predicted by competitive dynamics. The result corroborates an earlier literature on price-value deviations that stretches from Shaikh and Ochoa to Tsoulfidis (2008, 2022), now confirmed on the largest available country sample.

This empirical regularity is not trivial. The SMD theorem establishes that neoclassical microeconomics cannot predict such regularity at the aggregate level. The classical prediction of turbulent equalization—prices gravitating around values with competitive noise, at a rate determined by capital mobility and industry-level profit rate differentials—is a genuine empirical prediction that the neoclassical framework cannot generate.

The quantitative scale of international value transfers implied by this framework has been established by a major programme of empirical research. Hickel et al. (2022), using environmental input-output data and footprint analysis, estimate that in 2015 the global North net-appropriated from the South 12 billion tons of embodied raw material equivalents and 21 exajoules of embodied energy through price differentials in international trade. Their subsequent study (Hickel et al. 2024) — extending the analysis through 2021 using a comprehensive multi-country dataset accounting for skill levels, sectors, and wages — finds that in 2021 Northern economies net-appropriated 826 billion hours of embodied labour from the South, with a wage-value equivalent of €16.9 trillion at Northern prices, despite Southern wages being 87–95% lower than Northern wages for work of equal skill. These magnitudes confirm that the non-conservation of value at the international level is not a marginal theoretical qualification but a structural feature of the world economy of the first order. Ricci (2018) provides a complementary theoretical account in a disaggregated monetary model, demonstrating all forms of unequal exchange from Marx's LTV on a fully consistent basis.

8.2 Differential Organic Composition and Value Transfer

At the international level, however, the conservation of value encounters a structural asymmetry. Işıkara and Mokre's theoretical contribution (in *Marx's Value Theory at the Frontiers*, 2026, Ch. 4) operationalises the Emmanuel–Mandel–Shaikh mechanism of unequal exchange: countries with higher *value composition of capital* (VCC) transfer value to trading partners even at equal exchange prices, because vertically integrated labour contents differ from price ratios.

The mechanism is precise. Define the VCC of country k as $\kappa_k = C_k/V_k$—the ratio of constant to variable capital in the aggregate. At equal profit rates, prices of production for high-κ sectors contain less new labour relative to total price than for low-κ sectors. When high-VCC (core) and low-VCC (periphery) countries trade at prices equalising profit rates, the core country's commodities contain more embedded labour per monetary unit of price than their prices reflect—they sell "above value"—while peripheral commodities sell "below value." This is value transfer mediated by price.

In the Hamiltonian framework, this corresponds to a non-zero $\dot{H}$ at the *global* level: the international exchange system is not conservative. The value potential "drains" from the periphery to the core through the price mechanism, even when all individual exchanges are formally equal. The global Hamiltonian decreases in the periphery and increases in the core—precisely the pattern Işıkara and Mokre identify with empirical data on embodied labour flows across the WIOD country sample.

Skill-adjusted labour (I&M 2021's correction $\ell'_j = (1/\bar{w})(w_j/X_j)L_j$) addresses the further complication that heterogeneous concrete labour must be reduced to abstract labour equivalents. The skill-wage adjustment—weighting sectoral hours by relative wages—is the empirical implementation of Marx's theoretical reduction of skilled to unskilled labour, mediated by the social valuation of different types of labour-power. Without this correction, cross-country value comparisons confound the production of value with the realisation of value, precisely the confusion Rotta's framework risks.

8.3 Rotta and Complexity Rents

Rotta and Kumar (2023) approach international value transfers from a different angle: the productive/unproductive distinction applied globally, with *knowledge rents* and *complexity rents* as the value-capture mechanism for wealthy economies. Complex economies (high ECI rankings) systematically capture more value-added than they produce in labour terms; the gap is attributed to the commodification of knowledge, which has near-zero reproduction cost and hence generates rent-like revenues.

The Rotta framework and the Işıkara–Mokre framework are not contradictory but complementary: they address different sides of the same phenomenon. Rotta identifies the distributional outcome (value-added realization exceeds value-added production in complex economies) and its correlates (economic complexity, productive/unproductive classification).

Işıkara and Mokre identify the *structural mechanism* generating this outcome (differential VCC and rate of surplus value creating systematic price-value deviations in traded goods).

The integration of these frameworks within the Hamiltonian perspective suggests a richer typology of international non-conservation:

1. **Organic composition channel** (I&M): value transfer through traded goods prices reflecting differential organic compositions—a structural feature of the technology gap between core and periphery.
2. **Knowledge rent channel** (Rotta): value transfer through the monopoly pricing of commodified knowledge—a feature of financialised intellectual property regimes that emerged after the 1970s.
3. **Ecological rent channel** (I&M Book, Ch. 5): value transfer through resource extraction pricing—a Ricardian rent arising from the scarcity of natural resources.

Each channel corresponds to a distinct mode of $\dot{H} \neq 0$ at the international level. The organic composition channel is ancient and structural; the knowledge rent channel is historically specific to the era of intellectual property monopolies; the ecological rent channel is growing in importance as resource scarcity intensifies.

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9 Ecological Ground Rent as Terminal Dissipation

9.1 The Price-Value Signature of Ecological Scarcity

Işıkara and Mokre's ecological extension (*Marx's Value Theory at the Frontiers*, Ch. 5) uses EXIOBASE 3.8.2 environmental satellite accounts (footprint data, 1995–2020) to show that resource-extracting sectors—energy, mining, agriculture—are systematic outliers in price-value deviation analysis. Their prices consistently exceed both direct prices and prices of production, signalling a systematic rent component that cannot be explained by competition alone. This finding acquires greater weight against the backdrop of the Hickel et al. (2022, 2024) results: ecological resource flows are one of the primary vehicles through which the North appropriates embodied labour and materials from the South, and their pricing at market premiums above labour-content prices is the economic expression of this appropriation.

9.2 Ground Rent in the Sraffian Framework

The classical theory of ground rent provides the theoretical key. Ricardian rent arises because natural resources are non-reproducible: unlike manufactured commodities, they cannot be produced in any desired quantity at will. The scarcity of a resource imposes an additional charge above its price of production, appropriated by whoever holds property rights over the scarce natural condition. This rent is not value created by labour; it is value transferred from the rest of the economy to the resource-controlling sector.

Ehara (2023) provides the rigorous Sraffian formalisation of this mechanism, reconstructing Marx's theory of ground rent through price equations in the tradition of the Japanese Koza and Uno schools. Using a two-sector model, Ehara demonstrates how the landowner as a third class extracts rent through price formation, and — crucially — how intellectual property rights can function as a structurally analogous rent-extracting mechanism in knowledge-intensive sectors, crowding out land-based rent in other parts of the economy. This Sraffian rent model, in which rent appears as a price premium above the competitive price of production enforced by non-reproducibility, maps directly onto the ecological rent premium visible in I&M's price-value deviation data. Pirgmaier (2020) has argued more broadly that Marxian value theory is the only theoretical framework in economics that provides a *systemic* critique of capitalism adequate to ecological analysis, precisely because it grounds exchange values in production relations rather than in preferences or scarcity signals alone.

In the Hamiltonian framework, ground rent is a permanent source of $\dot{H} < 0$ for non-resource sectors: they continuously transfer value to resource sectors via price premiums. The global value system is therefore doubly non-conservative: value flows from periphery to core through differential organic composition, and value flows from all producers to resource-controllers through ground rent. These two channels interact: peripheral economies typically depend more heavily on resource extraction and thus face both organic composition disadvantages and the internal extraction of ground rent from their own productive sectors. Moore (2011) captures the dynamic character of this process in his world-ecology framework: capitalism does not merely act *upon* nature but develops *through* nature-society relations, with each frontier of accumulation producing a metabolic rift that eventually forecloses itself and requires a new frontier of appropriation.

9.3 The Metabolic Rift as Entropy Accumulation

The ecological crisis intensifies this dissipation in a way that has no finite resolution. As resource extraction approaches ecological limits—as marginal extraction costs rise, as ecosystem services are degraded, as climate instability increases production costs—the rent component in resource prices grows. This is a thermodynamic process in the most literal sense: the global value system approaches a state of increasing entropy in which the degradation of natural conditions of production cannot be offset by any expansion of the catalytic production network.

The metabolic rift concept, developed from Marx's analysis of the disruption of nutrient cycles in 19th-century agriculture and elaborated by Foster, Burkett, and Saito (2020), expresses in ecological terms what the Hamiltonian framework expresses mathematically: the capitalist production system is an open dissipative system that maintains its internal organization by continuously degrading the external natural conditions on which it depends. The Luxemburgian thesis about external markets has a precise ecological analogue — when no new ecological frontier remains to enclose, the entropy sink disappears and the system confronts its thermodynamic limits directly. The metabolic rift is not merely an environmental problem superimposed on an otherwise functional economic system; it is the ecological expression of the growing non-conservation of the global value Hamiltonian.

9.4 A Critical Response to Hornborg

Hornborg (2020) has mounted an influential challenge to this entire framework, arguing that the LTV is an *economic* argument incapable of accounting for asymmetric *biophysical* resource flows. He contends that market pricing leads to asymmetric resource flows regardless of any theory of labour value, and that attempting to express the metabolism of the world-system in monetary terms misses the physical reality of what is being transferred. In Hornborg's view, both LTV and neoclassical economics share a common failure: the assumption that commodities have objective values expressible in a common metric.

This challenge is serious and demands a response. We accept Hornborg's premise that biophysical flows (tons of raw materials, exajoules of energy, hectares of land) cannot be fully reduced to labour-time: the ecological throughput that sustains production has no labour-value equivalent in any strict sense. But we reject the conclusion that LTV must therefore be abandoned. The Hamiltonian framework allows us to distinguish *two levels* of analysis: (i) the conservative dynamics of value circulation within the production network, governed by labour-time and the MELT; and (ii) the non-conservative dissipation at the boundary between the production network and the natural world, governed by thermodynamic scarcity and expressed as ground rent. Ground rent is precisely the category that Marx developed to handle non-reproducible inputs — inputs whose price cannot be explained by the labour-time required for their production because no such labour is required. It is the *point of articulation* between the value system and the physical limits of nature, not a failure of the value system. The empirical manifestation — systematic price-value deviations in resource sectors — is the economic signature of this articulation, visible in I&M's EXIOBASE data and confirmed by Hickel et al.'s biophysical flow analysis.

The empirical signature of this process in I&M's data is clear: energy and mining sectors show the largest and most persistent positive deviations of market prices from prices of production—not because of monopoly power per se, but because of the rent component that competition cannot arbitrage away. The EXIOBASE footprint data allow I&M to connect this price-value pattern to physical ecological throughput: economies with larger ecological footprints per unit of labour embodied in their trade exhibit larger price-value deviations, consistent with the ground rent mechanism. Far from refuting the LTV, this pattern demonstrates its capacity to identify, at the level of prices, the imprint of thermodynamic boundaries on economic flows.

10 Towards a Unified Framework: Conservation, Dissipation, and the Critique of Political Economy

The foregoing analysis allows us to synthesise the main threads into a unified account that responds to Mirowski's challenge.

The *conservation condition*—value conserved in exchange, created in production through living labour—holds precisely as the aggregate identity mediated by the MELT:

$$\mu = \frac{p^\top (I - A)x}{\ell^\top x}, \qquad p^\top (I - A)x = \mu \ell^\top x.$$

This is not a tautology. It is a contingent empirical relationship whose validity was confirmed by Işıkara and Mokre's turbulent equalization results—prices do gravitate around values with bounded deviations—and whose violation generates predictable directional flows. It is more like Newton's first law (conservation of momentum in the absence of external forces) than like an accounting identity: the conservation holds in the absence of structural asymmetries, and its violations are informative about the nature of those asymmetries.

The *Hamiltonian formalism* provides the dynamical expression of this conservation in multi-sector reproduction. Simple reproduction corresponds to $\dot{H} = 0$: value circulates conservatively among sectors. Expanded reproduction corresponds to $\dot{H} > 0$: living labour acts as a source term increasing the Hamiltonian level. Crisis corresponds to $\dot{H} < 0$ (local or temporary): financial dissipation, disproportionality, and devaluation destroy value that cannot be realised. International exchange introduces a persistent $\dot{H} \neq 0$ gradient between core and periphery. Ecological rent introduces a growing $\dot{H} < 0$ drain on global productive sectors.

The *Cambridge controversies* establish the negative case: the neoclassical programme cannot construct a consistent invariant because capital cannot be measured independently of distribution. The Sraffian standard commodity—the classical tradition's response to Ricardo's problem—provides the positive construction of an invariant measure of surplus. The MELT is the dynamic generalisation of the standard commodity: where Sraffa's construction identifies the equilibrium proportions of the value field, the MELT quantifies its rate of change through time.

The *theories of crisis* (Bukharin's disproportionality, Luxemburg's underconsumption, the falling rate of profit) are not competing theories but complementary descriptions of the different modes by which $\dot{H} \neq 0$ is generated: structural disproportions between departments, entropy gradients requiring external sinks, and the tendential displacement of living labour by constant capital. The MELT provides the common metric: a rising organic composition implies falling μ for a given nominal wage share—the rate of profit falls because the denominator $\ell^\top x$ declines relative to the total capital advanced.

Finally, the *international and ecological extensions* (Işıkara and Mokre, Rotta) are not peripheral additions to the theory but its necessary completion. A closed-economy theory of value conservation is internally coherent but empirically incomplete: the actually existing world capitalist system operates through value transfers that systematically violate conservation at the national level. The Hamiltonian framework, extended to the world economy, identifies these violations as structural features of the international division of labour—not random noise but systematic dissipation driven by differential organic compositions, knowledge monopolies, and resource scarcity.

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11 Conclusion

This paper has argued that the classical research programme in political economy—when reconstructed through the SS-LTV, the Hamiltonian formalism of Goodwin and Flaschel–Semmler, the Cambridge controversies, and extended via Işıkara–Mokre and Rotta—provides the most rigorous available realisation of Mirowski's demand for a genuine economic conservation law.

The neoclassical failure was decisive and complete: no utility-based invariant survives the integrability constraints, the SMD theorem, or the capital measurement problem. The classical success is conditional and incomplete in a different sense: conservation holds in the closed economy under simple reproduction, but the actually existing capitalist world system generates systematic violations of conservation through class relations in production (the rate of exploitation), competitive redistribution through prices of production (the transformation problem, resolved via MELT), international exchange asymmetries (differential organic compositions and knowledge rents), and ecological scarcity (ground rent). Each violation is directional, measurable, and theoretically explicable.

The key conceptual move is to distinguish between *levels* of conservation. At the level of total value in the net product (MELT conservation), the classical system is rigorously conservative. At the level of individual sectors, firms, or nations, systematic deviations arise that are themselves the object of scientific explanation—not anomalies that refute the conservation principle but manifestations of the forces that drive value circulation. This is precisely the structure of conservation in physics: the total energy of a closed system is conserved; the sub-system energies change, and those changes are the physics.

The implications for contemporary political economy are direct. The ongoing debates between Işıkara–Mokre and Rotta on international value transfers are not merely empirical disagreements; they reflect different theoretical commitments about the *mechanism* of non-conservation. Integrating these frameworks—turbulent equalization at the micro level, complexity rents at the sector level, differential organic composition at the structural level, and ecological rent at the natural boundary—requires precisely the kind of unified Hamiltonian theory that the present paper has sketched. It is a research programme, not a finished theory; but its foundations are now substantially clearer than they were when Mirowski posed his challenge.

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12 Appendix A: The Goodwin Hamiltonian

The Goodwin (1967) model:

$$\dot{\lambda} = \lambda(a - b\omega), \qquad \dot{\omega} = \omega(-c + d\lambda),$$

is a Lotka–Volterra system with employment λ and wage share ω. Setting $q = \ln \lambda$, $p = \ln \omega$:

$$\dot{q} = a - be^p = \frac{\partial H}{\partial p}, \qquad \dot{p} = -c + de^q = -\frac{\partial H}{\partial q},$$

where $H(q,p) = ap - be^p + cq - de^q$. Direct verification:

$$\frac{dH}{dt} = \frac{\partial H}{\partial q}\dot{q} + \frac{\partial H}{\partial p}\dot{p} = (c - de^q)(a - be^p) + (a - be^p)(-(c - de^q)) = 0.\checkmark$$

The orbits are level sets $H = \text{const}$, representing perpetual redistribution of value between wages and profits with no net creation or destruction.

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13 Appendix B: The Flaschel–Semmler Hamiltonian and Non-Conservative Extension

Let $A \in \mathbb{R}^{n \times n}$ be the input-output matrix, B the output matrix, R a uniform profit factor, and $M = B - RA$. Define $y = \ln p \in \mathbb{R}^n$ and $z = \ln x \in \mathbb{R}^n$ with $\Delta_p, \Delta_x > 0$ diagonal. The adjustment system is:

$$\dot{y} = -\Delta_p M e^z, \qquad \dot{z} = \Delta_x M^\top e^y.$$

Proposition. When $\Delta_p = \delta_p I$ and $\Delta_x = \delta_x I$, the system is Hamiltonian with:

$$H(z,y) = \delta_x \mathbf{1}^\top M^\top e^y + \delta_p \mathbf{1}^\top M e^z.$$

Proof. $\frac{\partial H}{\partial y} = \delta_x M^\top e^y = \dot{z}$ and $-\frac{\partial H}{\partial z} = -\delta_p M e^z = \dot{y}$. Then $\dot{H} = \nabla_y H \cdot \dot{y} + \nabla_z H \cdot \dot{z} = \delta_x (M^\top e^y)^\top (-\delta_p M e^z) + \delta_p (M e^z)^\top (\delta_x M^\top e^y) = 0$. $\square$

Non-conservative case. When Δ_p and Δ_x are arbitrary positive diagonal matrices:

$$\dot{H} = (e^y)^\top M \Delta_x M^\top e^y \cdot \mathbf{1} - (e^z)^\top M^\top \Delta_p M e^z \cdot \mathbf{1}$$

which may be positive (value creation/dissipation favours quantity adjustment) or negative (price adjustment dominates). This signed quantity measures the net *sectoral power flow* in the system—the rate at which structural asymmetries inject or withdraw value from circulation.

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14 Appendix C: The MELT, SS-LTV, and the Two Marxian Equations

Let A be the $n \times n$ input matrix, $\ell \in \mathbb{R}^n$ the row vector of labour coefficients, $p \in \mathbb{R}^n$ the price vector, $x \in \mathbb{R}^n$ the activity vector.

Values: $\lambda = \lambda A + \ell$, so $\lambda = \ell(I - A)^{-1}$ (row vector of total labour contents).

MELT: $\mu = p^\top(I - A)x/\ell^\top x = p^\top(I - A)x/\lambda(I - A)x \cdot \lambda(I - A)x/\ell^\top x$.

First Marxian equation (SS-LTV): $p^\top(I - A)x = \mu\ell^\top x$ — the price of net product equals the monetary expression of living labour. This holds by definition of μ.

Second Marxian equation: $\Pi = \mu e^\top x$ where $e = \ell - \ell_v$ is the surplus labour vector and $\ell_v = \lambda w/\mu$ is the necessary labour (value of labour-power defined as UPP — unallocated purchasing power). Then $\Pi = \mu(\ell - \ell_v)^\top x = \mu e^\top x$. This holds for *any* price system, not just prices proportional to values: total profit equals the monetary expression of total surplus labour. The transformation problem is dissolved: there is no second constraint to be violated.

The wage-profit frontier: From the Sraffian standard system with standard commodity q satisfying $Aq = q/(1 + R)$:

$$r = R\left(1 - \frac{w}{\mu}\right),$$

where R is the maximum profit rate, w the money wage, and μ the MELT evaluated at standard commodity prices. The trade-off is linear, confirming Ricardo's search for an invariable measure of value: in the standard system, μ is independent of distribution.

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